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AN EXPLANATION OF THE FALSE SPECTRA FROM DIFFRACTION GRATINGS.

By Theodore Lyman.

With a Plate.

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AN EXPLANATION OF THE FALSE SPECTRA FROM DIFFRACTION GRATINGS.

By Theodore Lyman.

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In a previous paper * the author has shown that the principal spectra produced by concave diffraction gratings are complicated by the presence of false spectra of lower order than the first, and that these false spectra are common and often of excellent definition. It is the purpose of this paper to show that these false spectra are not of the nature of "ghosts," and that, while the theories developed to explain the latter cannot be made to fit them, a theory proposed by Professor Carl Runge, after examining one of the plates which formed the basis of the earlier paper, furnishes an explanation of the phenomena.

The "ghost," so called, is a faint reproduction of a real line, and in general occurs within a few Ångström units of its parent. The false spectra not only occur in regions many hundred units from that occupied by the principal line, but also differ fundamentally from "ghosts" in other respects.

Mr. C. S. Pierce † has made a careful mathematical study of the subject, and his paper also contains experimental data on the position of "ghosts," showing a good agreement between the theory and the observed facts. His treatment, however, deals only with "ghosts" occurring very near the parent line. Rowland ‡ has given a theory, the formulae of which may be extended to the case of false lines occurring at a considerable distance from the parent. The author, however, has never been able to fit the positions or intensities of any of his observed false spectra into these formulae in the form given by Rowland. In the case of most gratings this does not seem surprising, since the false spectra are very numerous, of small intensity, and with wave-lengths which bear no simple

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* These Proceedings, **36**, No. 14.

† American Journal of Math., **2**, 330 (1879).

‡ Rowland's Physical Papers, p. 525.

relation to one another. The error in ruling which produces them must be of a very complex nature, and we cannot expect the phenomenon to be readily brought under mathematical analysis. In some cases, however, an exact treatment should be possible, for with certain gratings some few of the false spectra are very much stronger than the others. Here it is fair to suppose that we have a comparatively simple error combined with a complicated one. Under the circumstances it seemed profitable to attempt to modify the ordinary treatment as given by Rowland.

A grating of N lines may, if $N = n\,m$, be considered as a grating of m groups, each consisting of n lines. The spectrum of the first order may be considered as the spectrum of the nth order of a grating of m groups. Each group is in itself a grating of n lines. The intensity I_n, any point of a spectrum of a grating of n lines, is known to be proportional to $A^2\left(\frac{\sin n\,a}{\sin a}\right)^2$, where $a = e\,(\sin i - \sin r)\frac{\pi}{\lambda}$. A^2 is the intensity of one line, which may be different in different directions, and e is the grating space or distance between two consecutive lines.

If we consider the grating of m groups, the intensity at any point of its spectrum will be proportional to $I_n\left(\frac{\sin m\,a'}{\sin a'}\right)^2$, where $a' = n\,a$. The 1st, 2d, 3d. . . . . . $n - 1$ order will vanish for any value of A, because I_n vanishes for $a = \frac{\pi}{n}, \frac{2\,\pi}{n} \ldots\ldots \frac{(n-1)\,\pi}{n}$.

Now let us assume that in the ruling of n consecutive lines there is an irregularity, such that the grating of n lines which we take to be repeated m times is of itself an imperfect grating. Then I_n will not vanish at all places where $a = \frac{\pi}{n}, \frac{2\,\pi}{n} \ldots\ldots \frac{(n-1)\,\pi}{n}$ and some of the first $n - 1$ orders of the grating of m groups will be visible. So far this is the ordinary theory of false lines.

Further, let us assume that the grating of n lines has a periodic error. To make the matter definite, let us take this error as occurring every third line. That is to say, every third line is slightly out of place, while the other lines remain in their correct position. Then it is probable that I_n will have some appreciable value at $a = \frac{\pi}{3}$ and at $a = \frac{2\,\pi}{3}$. If we take the error to be somewhat irregular, the intensity I_n will spread to both sides of those positions where $a = \frac{\pi}{3}$ and $a = \frac{2\,\pi}{3}$. The intensity

of the light given by the whole spectrum for the wave-length λ will be appreciable for those values $a = \frac{\pi}{n}, \frac{2\pi}{n}, \ldots\ldots \frac{(n-1)\pi}{n}$ which come near to the values $a = \frac{\pi}{3}$, $a = \frac{2\pi}{3}$. Here n may be taken at pleasure. For the sake of a definite case, let $n = 70$. Then I_n will have an appreciable value at $a = \frac{23}{70}\pi, \frac{24}{70}\pi, \frac{46}{70}\pi, \frac{47}{70}\pi$; for $\frac{23}{70}$ and $\frac{24}{70}$ are the nearest values to $\frac{1}{3}$: $\frac{46}{70}$ and $\frac{47}{70}$ are the nearest values to $\frac{2}{3}$. Moreover, since $\frac{1}{3}$ of 70 is 23.33, the 23d spectrum will be nearer this position than the 24th, and therefore the stronger of the two. Similarly, as $\frac{2}{3}$ of 70 is 46.66, the 47th spectrum will be stronger than the 46th. Therefore, taking the number 70, the two spectra which are the stronger will be the lower of the first pair and the higher of the second.

If, then, we let $n = 70$, and consider the error to occur every third line in the grating, for each line in the spectrum there will be four repetitions between its normal position and the direct image of the slit. These repetitions correspond to the 23d, 24th, 46th, and 47th order of the grating of m groups, the 70th order corresponding to the normal position.

This simple treatment of false spectra was suggested to the author by Professor Runge. The values used are those which seem most nearly to fit the case of the false lines obtained by the author from the grating called No. I, and illustrated in a former article.

Before proceeding further to the numerical verification of this theory, it may serve to illuminate the matter if we place in contrast with it the results which may be expected from the theory of Rowland.* The theory of Rowland is divisible into two parts, one dealing with the production of ordinary ghosts, the other part dealing with the production of lines at a considerable distance from the parent. It is this second part which could alone be expected to fit the case in hand. It does not do so, however, for it demands lines whose apparent wave-lengths bear simple ratios to the parent line. Professor Runge's theory shows the possibility of the total absence of lines at these positions indicated by Rowland, and it shows the probability of the formation of lines on either side of these positions at different distances and of an indicated relative intensity. Thus, in the present case, the theory of Rowland might be made to call for lines at positions corresponding to $\frac{2}{3}$ and $\frac{1}{3}$ the wave-length of the parent line. The lines actually observed do not fulfil this condition, but occur in flanking positions. For example, in the spark spectrum of

* Rowland's Physical Papers, p. 535.

magnesium, which is illustrated in Plate 1, the line of wave-length 2790.8,* which forms the most refrangible member of a characteristic group, is seen to be reproduced four times. Two of these false spectra are in the region between wave-lengths 1800 and 1900 and two in the region between 900 and 1000. It is to be noted that of the two groups near 1800 the less refrangible is the stronger, while of the groups near 900 the more refrangible is the stronger. This is in accord with the theory, for if the two strong reproductions are considered to be the 47th and 23d spectra of the grating of *m* groups, the two weak reproductions are the 46th and 24th spectra, while the real line 2790.8 corresponds to the 70th spectra.

When, however, the wave-lengths as measured in the previous paper were compared with those calculated from the fractions $\frac{47}{70} \times 2790.8$, $\frac{46}{70} \times 2790.8$, $\frac{24}{70} \times 2790.8$, $\frac{23}{70} \times 2790.8$, the observed facts did not seem to agree with the theory as accurately as might be expected. It seemed worth while to remeasure the wave-lengths of the false spectra, in order to determine if the fault lay in the theory or in the observed values. The author was encouraged in this step by the interest which Professor Runge took in the matter. In fact the method employed in re-measurement was suggested by him.

The arrangement of the apparatus was as follows: —

Two slits, *A* and *B*, were placed upon the circle whose diameter was the radius of the concave grating. The grating itself was kept fixed in position and the normal to its surface fell midway between the two slits. By this method the image of *A* was formed at *B*. An arm carrying the plate-holder *C* was pivoted at the centre of the circle. The result of this plan was that the first spectrum obtained when *A* was used as source, was shifted with respect to that obtained with *B* as source by a definite amount. The heights of the two slits were so arranged that the *A* spectrum fell directly over the *B* spectrum upon the photographic plate.

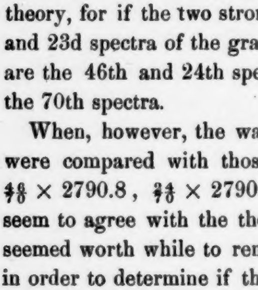


FIGURE A.

The method of procedure was to illuminate *A* with the light from a magnesium spark and to place the plate-holder *C* in such a position that a photograph of the false lines under investigation was obtained. Next

* Exner and Haschek, K. Akad. der Wiss. in Wien, **106**, Abth. II. (1897).

B was illuminated by light from a spark between iron terminals, and thus the standard iron spectrum was photographed under the false magnesium lines. To determine in Ångström units the amount by which the one spectrum was shifted with respect to the other, both *A* and *B* were illuminated with light from the iron spark and the relative position of two known lines was measured. As an example of the method, the false line, which by the theory should be the 47th order of the grating of *m* groups, was compared with the line in the iron spectrum 3541.2 * and found to have the relative position 3541.0. Next, the line 2737.0 * in spectrum *A* was compared with the line 4404.9 * in spectrum *B*. The difference between them was .5 Ångström units. Thus the shift of spectrum *A* with respect to *B* was $[(4404.9 - .5) - 2737] = 1667.4$, and the apparent wave-length of the false line was $3541.0 - 1667.4 = 1873.6$. In this way the apparent wave-lengths of the four false lines were determined. In order to check the accuracy of the method, slit *B* was moved toward slit *A* and the grating was re-adjusted. In this way the shift of one spectrum with respect to the other was altered and the unknown spectra were compared with a new position of the iron spectrum. The following table gives some idea of the accuracy of the method and of the agreement between the observed and the calculated values.

Calculated Wave-length.	Measured Apparent Wave-length.	
	First Observation.	Second Observation.
917.0	917.0	916.8
956.8	956.7	956.4
1834.0	1834.1	1833.9
1873.8	1873.7	1873.7

The method gives an average error of 0.17 of an Ångström unit. The average difference between the observed and the calculated values is .14 units. Thus the difference between theory and practice is within the errors in observation.

It might seem that values different from $n = 70$ and a period of 3 would satisfy as well the conditions when substituted in the equations.

* Exner and Haschek, K. Akad. der Wiss. in Wien, **106**, Abth. II. (1897). Compare also Kayser and Runge Arc Spectrum.

This is not the case, as may be shown by trial. If there are to be but four reproductions and if the relative dispersions and intensities of these reproductions are to be explained, the values 70 and 3 give the result best in agreement with experiment.

Up to this point it has been the object of the discussion to contrast, in the light of these experiments, the theory of Professor Runge with that part of the theory as given by Rowland which would seem most obviously applicable to the case. This part of the theory was called "One line in *m* displaced.*" The other part of the analysis which deals with "ghosts" came under the title "Periodic Error."† A moment's consideration will show that the lines under discussion are not of this latter class.

In the case of Simple Periodic Error the position of any groove in the grating ruling is given by the equation, $y = a_0 n + a_1 \sin (e n)$. Thus the *n*th groove from a fixed line of reference is out of its true position by a term which varies as a sine function with period e; the maximum value of the displacement is of course a_1. Thus no groove in the grating surface is exactly in its proper place unless $\sin (e n) = 0$. The system of ghosts to which this form of error gives rise is characterized by the following properties.‡ Ghosts of any order must occur in pairs. Of a pair one lies to the right of the parent line, the other to the left. Ghosts of the second order must lie exactly twice as far from the real line as ghosts of the first order; ghosts of the third order three times as far. The distance of a ghost of a given order from its parent line is a constant independent of the order of the spectrum in which the parent line is measured. These three properties are not in any way possessed by the lines under discussion. This, together with the fact that no numerical application of the theory of ghosts to the case in hand seems possible, excludes it from further consideration.

In short, then, Professor Runge's explanation of the false spectra seems to fit the facts most accurately. It is perfectly possible to extend the theory to even the more complex cases where there are more than four reproductions of every real line. The period of the error and the value of *n* may be taken at pleasure, so that the treatment can be made to fit a great variety of cases. In practice, however, the errors of ruling in those gratings which give a great number of faint false spectra are too complex to make calculation profitable.

It may be of interest to remark that false spectra are not confined to concave diffraction gratings, but are to be found in the spectra produced

* Rowland's Physical Papers, p. 535. † Ibid., p. 536. ‡ Ibid., p. 519.

by plane reflection gratings as well. The author has examined two instruments ruled upon speculum metal by Rowland's engine. In the experiment the light was collimated and brought to a focus by quartz lenses. In order to reduce the phenomena to the simplest possible form, a line in the visible blue spectrum of magnesium was separated out by a prism spectroscope and thrown upon the slit of the collimator. The camera was focussed on the image of this line in the first spectrum. The first grating was ruled in 1883 and had 14,438 lines to the inch. The image of the line used appeared in its proper place in the first spectrum — = 4481 Ångström units — and was not accompanied by any ghosts of the common kind. That is to say, there was no doubling of the line, nor were there any faint reproductions very near it. In the region near wave-length 3000, however, four sharp lines occurred; and again near wave-length 1550 four more reproductions were present. Thus this grating produces eight false spectra of a lower order than the first, corresponding to the real line 4481. Besides these eight, numerous faint reproductions may be detected, but they are of extremely feeble intensity.

The second grating examined was ruled in 1887 and had also 14,438 lines to the inch. The spectrum obtained with it, however, was very different from that given by the first instrument. The first spectrum of the line 4481 was, as before, sharp and without ghosts, but the eight distinct false spectra were replaced by at least seventy reproductions of very feeble, but nearly equal, intensity. These extended between positions corresponding to wave-lengths 2000 and 900.

The results obtained with these two plane gratings are exactly similar in character to those obtained with the concave gratings called No. I and No. II and recorded in a previous paper. These gratings seem to belong to two types, the one in which the false spectra are all of nearly equal intensity and feeble, the other in which some few of the false spectra are many times more intense than the others. In the one case the grating gives a background of faint lines; in the other sharp, strong false spectra are present.

The author wishes to call attention to the plate which accompanies this article. It is from a concave grating of 6-foot radius and shows the false lines whose positions have been discussed in this paper. The plate is taken directly from a negative by photographic process. The two groups at positions corresponding to wave-lengths 956 and 1834 are very faint in this reproduction. Their positions are indicated, however. The character and dispersions of the two stronger groups is well shown. All the lines in this plate are false.

The pairs of sharp lines in the immediate neighborhood of the characteristic groups to which the treatment has been confined are reproductions of the real lines at 2936.8 and 2928.9. They form the 47th, 46th, 24th, and 23d spectra of the system in which the real lines are the 70th spectra. The single line near the middle of the plate is the 23d false spectrum of the real line at 4481.3. The lines not far from it are the 23d false spectrum corresponding to the real lines 3838.5, 3832.5, and 3829.5. The 24th spectra of all these lines are too feeble to reproduce. Their 46th and 47th spectra occur among real lines in the region between 3050 and 2500. The apparent wave-lengths of these false lines are marked upon the plate; the decimal place is omitted. The following table gives the measured apparent wave-lengths of all the lines appearing upon the plate, their calculated wave-lengths, their order as false spectra, and the wave-lengths of the real lines of which they are the reproductions.

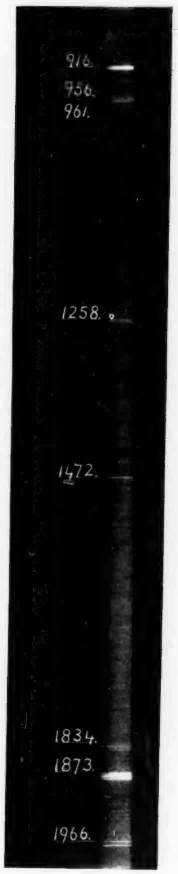
Apparent Wave-lengths.		Real Wave-lengths of Parent Lines.	Order as False Spectra.
Calculated.	Measured.		
917.0	917.0	2790.8	23d
956.8	956.7	2790.8	24th
962.3	961.8	2928.9	23d
1258.3	1258.4	3829.5	23d
1472.4	1472.8	4481.3	23d
1834.0	1834.1	2790.8	46th
1873.8	1873.7	2790.8	47th
1966.5	1966.5	2928.9	47th

It is well to remember that this grating is not unique, but that most reflection gratings produce false spectra. It is in the extreme ultraviolet that these false lines are most easily seen, and it is in this region that they may be most readily taken for real lines. The lines may serve to show by their strength and sharpness the danger which they offer in spectroscopic work.

In conclusion, it may be well to repeat the chief result of this paper. False spectra differ fundamentally in character from the commonly ob-

served "ghost." The former seem due to a so-called periodic error in the grating ruling, an error which operates to displace every groove in the grating surface by an amount depending on a sine function of the position of the groove. It seems probable that the false spectra are due to an error of another type. Here the error operates to displace one, out of a given number of grooves, slightly, leaving the remainder in their proper positions. In order to make theory and the observed facts agree, this error must be considered somewhat irregular over the surface of the grating. While the theories proposed by C. S. Pierce and Rowland account in every way for the phenomenon of ghosts, they do not either qualitatively or quantitatively account for these false spectra, whereas the theory proposed by Runge, and given above, explains the phenomena qualitatively and very nearly quantitatively. That is to say, it explains the production of lines far from the parent line, lying entirely on one side of it; it explains their relative intensity, and it explains very nearly indeed their exact position. The maximum departure between the positions of these false spectra demanded by the theory and observed in the plates is not more than .4 Ångström units. This is slightly greater than the maximum difference found between sets of observations; but it is to be remembered that the measurements depended upon the wave-lengths of the parent group and upon the comparison iron lines, both of which are borrowed data.

Jefferson Laboratory, Harvard
University, 1903.


916.
956.
961.
1258.
1472.
1834.
1873.
1966.